



Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at <http://about.jstor.org/participate-jstor/individuals/early-journal-content>.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

CONDITION IN TERMS OF THE INVARIANTS OF THE QUARTIC THAT ITS FOUR DISTINCT ROOT-POINTS BE CONCYCLIC.

By DR. T. E. MCKINNEY, Wesleyan University, Middletown, Conn.

The necessary and sufficient condition that the root-points of the quartic

$$(1) \quad \sum_{i=0}^{i=4} a_i z^{4-i}, \quad a_i \text{ complex,}$$

be concyclic is that an anharmonic ratio of the roots of the quartic be real. The equation giving the six anharmonic ratios of these four roots is

$$(2) \quad t^6 - 3t^5 + \left(6 - \frac{I^3}{\Delta}\right)t^4 - \left(7 - \frac{2I^3}{\Delta}\right)t^3 + \left(6 - \frac{I^3}{\Delta}\right)t^2 - 3t + 1 = 0,$$

where $a_0^2 I = a_2^2 - 3a_1 a_2 + 12a_0 a_4$;

$$a_0^3 I_1 = 27(a_1^2 a_4 + a_0 a_3^2) - 9a_1 a_2 a_3 - 72a_0 a_2 a_4 + 2a_2^3,$$

$$27a_0^6 \Delta = 64I^3 - I_1^2.$$

The discriminant D of equation (2) is

$$\begin{aligned} D &= \prod (t_i - t_j)^2, \quad i=1, 2, \dots, 5; j=i+1, \dots, 6, \\ &= \frac{I^1 2}{\Delta^4} \left(4 \frac{I^3}{\Delta} - 27\right)^3. \end{aligned}$$

Every root of equation (2) is a rational function with real coefficients of every other, so that the roots are either all real or all complex. When the roots are all real I^3/Δ is real. When the roots are not only real but also distinct, $D > 0$. When I^3/Δ is real, and $D > 0$, equation (2) has an even number of pairs of conjugate roots. Hence two and, therefore, all roots are real. This result may be expressed as follows:

THEOREM. *The necessary and sufficient condition that the four distinct root-points of the quartic*

$$\sum_{i=0}^{i=4} a_i z^{4-i}, \quad a_i \text{ complex,}$$

be concyclic is that $\frac{4I^3}{\Delta} - 27 > 0$, where I and Δ are the invariants of the quartic.